SUGGESTED REFERENCES: CHAPTER 2

R. COURANT and **D. HILBERT**, *Methods of Mathematical Physics,* vol. **1. The** literature on the calculus of variations is daunting in its volume and usually covers far more than is needed for the purposes of this chapter. Most treatises on "mathematics for the physicist" contain some brief discussion of the calculus of variations, and the classic work of Courant and Hilbert gives one of the clearest in their Chapter IV

R. WEINSTOCK, *Calculus of Variations.* This is a book a physicist can feel comfortable with. Indeed over half the book is concerned with applications to problems of physics. The fundamental lemma is treated at the start of Chapter 3 and some of the difficulties of the continuous solution to the problem of the minimum surface of revolution are described in Section 3—7.

G. A. BLISS, *Calculus of Variations.* Of the older literature on the calculus of variations this little book is notable for its detailed discussion of the nature of the solutions to some of the standard problems, such as the brachistochrone. Chapter 4 has all that one would possibly want to know about surfaces of revolution of minimum area.

L A. PARS, *An Introduction to the Calculus of Variations.* A careful and painstaking survey of the simpler mathematical aspects of the calculus of variations paying some attention (less than the author thinks) to physical applications. The fundamental lemma (and some cognate theorems) are presented in Chapter II. There is a brief note on nonholonomic systems on page 253.

C. LANCZOS, *The Variational Principles of Mechanics.* The first five chapters of this book are a leisurely survey, flavored by the author's original viewpoint, of the content of the present chapter. Chapter 10, containing historical notes, is particularly interesting. This is probably the best single reference for the entire chapter, although the approach differs considerably from that given here.

E. WHITTAKER, *Analytical Dynamics.* The treatment given for the topics in this chapter are still of interest, especially for many esoteric side notes not to be found elsewhere. Conservation theorems are discussed in Chapter III, while Hamilton's principle and its derivation from Lagrange's equations (the converse of the route taken in the present chapter) will be found in Chapter IX. The presence of a set of differential equations in the Lagrangian form thus always implies the existence of an associated variational principle. It is therefore of some interest to know when a set of differential equations of second degree are, or can be put, in the Lagrangian form. This problem was first tackled by Helmholtz in 1887. The conditions he found, and some associated consequences, are described in an admirable and detailed review paper by P. Havas in *Nuovo Cimento Supplement,* vol. 5, p. 363 (1957).

L. D. LANDAU and **E. M. LIFSHITZ,** *Mechanics.* This is the first volume of the *Course of Theoretical Physics,* that monument to the genius of Lev Landau. An incredible amount of material is contained within the brief compass of the 166 pages (in the English translation), and careful reading will be repaid in relation to almost any topic in the present book. The style might be described as that of 'hand-waving arguments" written down on paper, and some holes are often left in the reasoning, but the physical insights are invaluable. Chapters 1 and II are particularly relevant to the present chapter.

W.E. BYERLY, *Generalized Coordinates.* Happily still available in reprint form, this little book is of value particularly for the many detailed examples of the Lagrangian technique for setting up and solving mechanical problems. The lack of an index is a deplorable defect that makes use of the book somewhat difficult.

D. W. WELLS, *Lagrangian Dynamics.* Although sometimes mislabeled as unsophisticated, this "outline" contains a wealth of detail and practical problems on a wide variety of aspects of Lagrangian mechanics. Chapter 6 has an unusually detailed treatment of frictional and dissipative sources within the Lagrangian framework (the author's "power function" is our dissipation function). Chapter 15 is devoted to Lagrange's equations for electrical systems and their interaction with mechanical systems. Chapters 12 (on constraint forces) and 17 (on Hamilton's principle) are also useful.

H. F. OLSON, *Solution of Engineering Problems by Dynamical Analogies.* This book discusses in great detail the electrical circuit problems equivalent to given mechanical and acoustical systems and illustrates the application of circuit theory to the solution of purely mechanical or acoustical problems. Lagrangians per se are introduced and used only briefly. A more pervasive Lagrangian viewpoint characterizes the following reference.

B. R. GOSSICK, *Hamilton's Principle and Physical Systems.* Although other non-mechanical systems are discussed, the emphasis is on applications from electrical engineering. The author's particular interest is in nonconservative (but linear) systems, and he enlarges the concept of a dissipation function to include energy loss by electromagnetic radiation.

H. RUND, *Hamilton—Jacobi Theory in the Calculus of Variations.* A good deal has been written about Hamilton's principle for nonholonomic systems, and most of it is wrong (including some things that were said in the first edition). Rund's book, though highly mathematical, has a number of interesting discussions on "pathological" problems encountered in the actual physical world and will be referred to here on several occasions. What is particularly relevant here is Section 5.5 on nonholonomic dynamical systems, which arrives at the flat conclusion that Hamilton's principle, in the form of Eq. (2—2), is applicable only to holonomic constraints. The Lagrange multiplier procedure used here, based on varied paths constructed from virtual displacements, is gone into much greater detail in a pair of papers by H. Jeffreys and L.A. Pars, respectively, published in 1954 in the *Quarterly Journal of Mechanics and Applied Mathematics.* vol. 7, p. 335 and p. 338.