SUGGESTED REFERENCES: Chapter 5

L. BRAND, *Vector and Tensor Analysis.* J. W. Gibbs introduced dyads and dyadics, and the fullest exposition of this subject is in the classic work *Vector Analysis* by Gibbs and Wilson 1901). An extensive treatment of dyadics is to be found in the somewhat later treatise by Wills, *Vector and Tensor Analysis.* The texts on tensor and matrix analysis referenced in Chapter 4 above may be used for most of the pertinent material of the present chapter. Brand's book is added here because in addition to the standard material on tensors it has a good deal on dyadics in his Chapter 4 on linear vector functions. The last chapter provides an easily accessible brief introduction to quaternions.

K. SYMON, *Mechanics.* The pragmatic usefulness of dyadics is exemplified in Symon's treatment in his Chapter 10 of the properties of the inertia tensor, where a number of interesting examples are given. The relevant algebra of dyadics and tensors is developed explicitly.

E. J. ROUTH, *Dynamics of Rigid Bodies, Elementary and Advanced.* In the nineteenth century the dynamics of rigid bodies formed one of the main topics at the research frontier of mechanics. Routh's two-volume treatise, the latest edition of which appeared in the 1890s, forms a convenient and elaborate presentation of the achievements made in this field up to that time. For many topics it remains an almost unique source in English. Routh, a colleague of Maxwell, was a pioneer in the study of the stability of small oscillations, and his work there is still relevant today.

W. D. Macmillan, *Dynamics of Rigid Bodies.* While not recommended for a systematic study of rigid body dynamics, this work contains much material not readily available elsewhere. Chapter VII, in particular, has long and elaborate discussions of Poinsot motion, and of the motion of the heavy symmetrical top, including the explicit solutions in terms of elliptic functions. The chapter on the complex problems of rolling rigid bodies is also worthy of note.

A. GRAY, *A Treatise on Gyrostatics and Rotational Motion.* The product of World War I interest in gyroscopic devices, Gray's treatise represents the culmination of the British tradition in the field of rigid body dynamics. in a somewhat dense and mostly non-vectorial treatment it covers a wide range of topics, from wandering of the Earth's rotational poles to the theory of the boomerang and the operation of the diabolo. A more systematic discussion of many of the same areas will he found in the works of Klein and Sommerfeld.

F. KLEIN and **A. SOMMERFELD,** *Theorie des Kreisels.* This monumental work on the theory of the top, in four volumes, has all the external appearances of the typical stolid and turgid German "Handbuch." Appearances are deceiving, however, for it is remarkably readable, despite the handicap of being written in the German language. The graceful, informal style has the fluency and attention to pedagogic details characteristic of all of Sommerfeld's later writings. Although the treatment becomes highly mathematical at times, the physical world is never lost sight of, and one does not founder in a maze of formula. Although limited by the title to tops and gyroscopes, the treatise actually provides a liberal education in all of rigid body mechanics, with excursions into other branches of physics and mathematics. Thus, Chapter I discusses, among other items, Euler angles, infinitesimal rotations, and the Cayley—Klein parameters and their connections with the homographic transformation and with the theory of quaternions. The later notes to this chapter (in Vol. IV) discuss also the connections with electrodynamics and special relativity (quantum mechanics was still far in the future). By and large, Vol. I lays the necessary foundations in rigid dynamics and gives a physical description of top motion with little mathematics.

Volume II is devoted to the detailed exposition of the heavy symmetrical top, although there is also much on Poinsot motion, and it contains a summary of what was then known about the asymmetric top. The distinction between regular and pseudo-regular precession was first introduced here and the authors spend much time in examining the two motions, and the approach to regular precession. Many pages are given to a thorough demolishing of the popular or elementary "derivations" of gyroscopic precession. (The authors remark that it was the unsatisfactory nature of these derivations that led them to write the treatise!) There is a long discussion on questions of the stability of motion. Most of the treatment is based on the solution in terms of elliptic integrals and not merely on the approximate small nutation, as was done here.

Volume III is mainly on perturbing forces (chiefly friction) and astronomical applications (nutation of the earth, precession of the equinoxes, etc.). The discussion of the wandering of the earth's poles is especially complete for the time it was written, including an estimation of the effects of the earth's elasticity and the transport of atmospheric masses by the wind circulation.

Volume IV is on technical applications and is rather out of date by now.

F. KLEIN, *The Mathematical Theory of the Top.*. In 1896 Felix Klein gave a series of lectures at Princeton, the notes for which constitute this slim volume, recently reprinted along with some unconnected mathematical articles. Most of the book is concerned with highly abstract mathematical details of the theory, but the first Lecture provides a readable account of Cayley—Klein parameters. It is interesting to note that both in this work and in the larger treatise with Sommerfeld use was made of a four-dimensional non-Euclidean space in which time is the fourth dimension-—anticipating the use in special relativity by many years (see next chapter). However, the space was solely for mathematical convenience and no physical significance was intended.

A. SOMMERFELD, *Mechanics.* Sommerfeld's work with Klein about the top was one of his first publications, while this text, part of a famous series of *Lectures on Theoretical Physics,* was published more than forty years later, as one of the last of his writings. His interest in the top had apparently not diminished in that time and he devoted considerable space to qualitative discussions of a wide range of gyroscopic and top phenomena even to a page or two on the asymmetrical top. The thirty or so pages on the entire subject occupy almost all of the chapter on rigid bodies and practically farms an abstract of the larger work! The treatment is extensive, rather than intensive, and there is little detailed discussion.

V. D. BARGER and **M. G. OLSSON,** *Classical Mechanics, A Modern Perspective.* This intermediate level text is referenced because it is about the only book that contains even a brief description of what the "tippie-top" is and how it works (p. *254).* For pictorial evidence of how the tippie-top has fascinated royalty and the great of physics alike, see the photographs opposite p. 208 in *Niels Bohr,* edited by S. Rozental. To the references given on p.224 above may be added a paper by T. R. Kane and D. A. Levinson, *Journ. Applied Mech.* **45, 903 (Dec.** 1978), which presents a modern computer solution of the tippie-top accompanied by an extensive bibliography.

J. AHARONI, *Lectures on Mechanics.* A discursive set of essays on relatively isolated topics in mechanics, this reference is noteworthy for its extensive collections of diagrams. The carefully thought-out figures in Chapter 15—17 may be found to illuminate some sticky points about the inertia tensor, Poinsot motion, and gyrocompasses, among other items.
L. MEIROXJITCH, *Methods of Analytical Dynamics,* and **S. W. GROESBERG,** *Advanced Mechanics.* In modern technological applications of mechanics, the dynamics of a rigid body plays a central role, not only for such devices as gyroscopes, but also as a first approximation to systems that are not entirely rigid such as a space ship. The need to solve actual problems and not merely derive formulations imposes a perspective on the methods of mechanics that often contrasts with the viewpoint of the physicist. These two texts give an introduction to the modern methods needed for tackling engineering problems.

E. LEIMANIS, *Motion of Coupled Rigid-Bodies.* Russian applied mathematicians have given much attention to the general problem of the motion of one or more rigid bodies about a fixed point. Their efforts stretch from the days of Sonya Kovalevskaya in the 1880s to the present time. Much of their work is not accessible in English. The reference of Leimanis is a modern treatment of the motion of rigid bodies, as viewed by an applied mathematician with a full understanding of the Russian literature. Up to date mathematical techniques, such as use of Lie series, are incorporated.

W.H. MUNK and **G. J. F**. **MACDONALD,** *The Rotation of the Earth.* One of the most fascinating applications of the dynamics of rigid bodies is to the phenomena of the rotating earth—although the first step is to realize to what an extent the earth and its appurtenances deviate from a rigid body. The treatise of Munk and Macdonald provides a well-written, lucid introduction to these geophysical applications, which range from continental drift through ancient historical records of eclipses to questions of earthquake excitation of Chandler wobble. Unfortunately – or is it happily? – the treatment cannot be considered as definitive because the field is undergoing intense development and significant advances have been made since the 1960 publication date. But any study of the features of the earth's rotation would do well to start with Munk and Macdonald.

F. D. STACEY, *Physics of the Earth.* Chapter 2 provides a compact discussion of such topics as precession of the equinoxes and the Chandler wobble, with references to the modern literature.

W. WRIGLEY, W. M. HOLLISTER, and **W. G**. **DENHARD,** *Gyroscopic Theory, Design and Instrumentation.* Gyroscopic devices are at the heart of modern advances in inertial navigation. The published literature is considerable, and the unpublished report literature is even more voluminous (not to mention the substantial oral tradition amongst the practitioners of the art). This treatise, a product of the famous M.I.T. Draper Instrumentation Laboratory, gives a reasonably modern overview of the field starting from a discussion of rotation matrices and ranging to detailed blueprints of actual devices. A short review paper by Wrigley and Hollister, "The Gyroscope: Theory and Application," *Science* **149,** 713 (Aug. 13, *1965),* may prove illuminating.