SUGGESTED REFERENCES

R. COURANT AND D. HILBERT, *Methods of Mathematical Physics*. The Hamilton-Jacobi equation is an example of a first-order partial differential equation in many variables, and the mathematical characteristics of this type of equation are important (as has been noted) for the development of Hamilton-Jacobi theory. Volume 2 of Courant and Hilbert provide a painless (nearly) introduction into the relevant mathematics, first in an introductory section (§4) in Chapter 1 and then in a lengthy explicit discussion of Hamilton-Jacobi theory in Chapter 2.

C. CARATHÉODORY, *Calculus of Variations and Partial Differential Equations of the First Order*. For many years the recognized authority on first-order partial differential equations, the exposition in Vol. 1 of Carathéodory goes into greater depth than Courant and Hilbert and is especially elaborate in developing the theory of what are called "characteristics." The Hamilton-Jacobi equation is not referred to by that name, but it, and its relation to mechanics, dominates most of the book.

C. LANCZOS, *The Variational Principles of Mechanics*. The treatment here of Hamilton-Jacobi theory (in Chapter 8) emphasizes the foundations of the theory and its geometrical interpretations. There is considerable discussion of the necessary conditions for action-angle variables (referred to as "Delaunay's treatment") and their nature. Detailed applications are few.

D. TER HAAR, *Elements of Hamiltonian Mechanics*. The treatment of the Hamilton-Jacobi equation and its applications, including the action-angle variable approach to the Kepler problem, roughly parallels that given in the present chapter. Inevitably there are differences of emphasis and viewpoint, and the reader may benefit from comparing the two discussions.

M. BORN, *The Mechanics of the Atom*. Physicists had a brief interest in action-angle variables in the heyday of the older quantum theory, when it provided the "royal road" to quantization. Born's book was a product of that period and remains one of the best discussions of Hamilton-Jacobi theory and action-angle variables accessible to physicists. It is outstanding in the wealth of the applications it presents. Born's discussion of action-angle variables and related perturbation theory is the source for the discussions on these areas as found in many text books in mechanics, and the present book is by no means a complete exception. The reader should be cautious, however, in accepting the statements in Born's book about atomic stricture. Most of them are out of date.

A. SOMMERFELD, *Atomic Structure and Spectral Lines*. The exposition of Hamilton-Jacobi theory and action-angle variables to be found scattered through the text and appendices of this book is considerably less detailed than in Born. Probably for that reason it is oftern more readable. Especially noteworthy is the discussion of the connection between the number of systems of separation coordinates and the degeneracy of the motion. The evaluation of the integrals occurring in the Kepler problem by means of the theory of residues is explained in an appendix (and is also given in Born's book).

J. H. VAN VLECK, *Quantum Principles and Line Spectra*. The chapter of this work entitled "Mathematical Techniques" provides a quick survey of Hamilton-Jacobi theory and action-angle

variables, with an introduction into perturbation theory. Most of the rest of the book is of historical interest only. The caution applied to Born's book is equally valid here and holds almost as well for Sommerfeld's volume.

B. GARFINKEL, *The Lagrange-Hamilton-Jacobi Mechanics* in *Space Mathematics*, *Part I*. Action-angle variables had been used in celestial mechanics (although not under that name) long before there was any physics interest in them, and their use remains today the elegant way of approaching perturbation theory. This reference shows how the subject is viewed from the standpoint of celestial mechanics, in an essay that's brief and concise but eminently readable (a characteristic notoriously lacking in many treatises on analytical mechanics). In a scant 36 pages it covers the field from Lagrangian mechanics through perturbation theory but packs an incredible amount of material in the brief compass. Examples are the Staeckel conditions for separability (there is an obvious factor of 2 missing in Eq. (82)), Vinti's theorem, and the Delaunay elements. Some of the notation and conventions differ from those customarily followed in physics.

L. A. PARS, *A Treatise on Analytical Dynamics*. Three chapters cover the area from the Hamilton-Jacobi equation to action-angle variables, which allows for a leisurely treatment and a plenitude of examples. Unusual is the discussion on separable systems as a property per se, independent of the application to the Hamilton-Jacobi equation. Staeckel's condition is treated at length. Action-angle variables are not mentioned explicitly (although angle variables are) but the sections on multiply-periodic motion are extensive.

H. V. MCINTOSH, *Symmetry and Degeneracy* in *Group Theory and its Applications*, Vol. 2, ed. E. M. Loebl. As might be expected the connection between separability and degeneracy (and the symmetry properties of the system) come in for considerable discussion here. The argument that the space-filling properties of the orbit in nondegenerate systems militates against separability is gone into in some detail, and the consequences for a wide variety of systems are considered (including such unusual ones as the magnetic monopole). All in all, this is probably the best available reference on the degeneracy-separability-symmetry connection available in the 1970s.

L. BRILLOUIN, *Tensors in Mechanics and Elasticity*. This charmingly written book contains much information on a wide variety of topics, from differential geometry to the quantum mechanics of solids (of circa 1938). The motion of the surfaces of constant S in configuration space is presented in detail in Chapter VIII, and the connections linking classical mechanics, geometrical optics, and wave mechanics are thoroughly discussed in Chapter IX.

M. BORN AND E. WOLF, *Principles of Optics*. A standard reference on the application of Hamilton-Jacobi theory to geometrical optics is the rather formidable treatise by J. L. Synge, *Geometrical Optics*. Born and Wolf provide a more understandable introduction, with considerable attention paid to further extension to the Schrödinger wave equation. A chapter on the foundations of geometrical optics contains, among other material, a derivation of the eikonal equation for a vector field. The following chapter on the geometrical theory of optical imaging starts out, at least, with a Hamiltonian approach. Two appendices are of particular interest in the connection. The first is practically a short treatise on the calculus of variations, with emphasis on the Hamilton-Jacobi equation. The contents of the following appendix are clearly indicated by the title: "Light optics, electron optics and wave mechanics."