SUGGESTED REFERENCES: Chapter 6

H.JEFFREYS and B. *S.* JEFFREYS, *Methods of Mathematical Physics.* Diagonalization of a matrix or of a pair of positive-definite quadratic forms is a common topic in texts on mathematical methods for physicists. This particular reference is chosen because the pertinent discussion in Chapter 4 has obviously been written with the problem of small oscillations particularly in mind. A number of theorems are given about the properties of roots of the secular determinant that are helpful in actually finding the eigenfrequencies.

E. T. WHITTAKER, *Analytical Dynamics.* Chapter VII is on the theory of vibrations and gives the explicit proof that T and V can be diagonalized together, but the treatment of this point is not very clear. More valuable are the later sections of the chapter on the effects of constraints, and on vibrations about steady motion, which is discussed in considerable detail. Section 94 of Chapter VIII on vibrations in the presence of dissipative forces is fragmentary and restricted to two degrees of freedom.

D. TER HAAR, *Elements of Hamiltonian Mechanics.* Despite its title this book is by no means confined to Hamiltonian mechanics, and Chapter 3 on small vibrations assumes no more than Lagrangian mechanics. The treatment is distinguished by a matrix approach—
with all the elements of the matrices written out in detail. There is an abundance of examples, with useful diagrams of the normal modes of vibration. Some of the conclusions on the symmetry properties of vibrations of the linear triatomic molecule are questionable.

J. L. SYNGE and **A. SCHILD**, *Tensor Calculus.* Chapter 2 of this treatise provides an introduction to Riemannian spaces and associated metric tensors that is more than adequate for our purposes. In particular, they introduce immediately the differentiation into covariant and contravariant quantities, a distinction not needed here, though it is raised in Chapter 7.

L. D. LANDAU and **E. M. LIFSHITZ**, *Mechanics.* As would be expected, the subject of small vibrations is presented compactly but covers a lot of ground. Oscillations of the triatomic molecule, both linear and "bent," are considered in some detail. In the present chapter only linear oscillations are considered, and the parametric elements of the potential and kinetic energy matrices are assumed constant in time. Study of parametric excitation of oscillators, in which these elements vary in time, and of nonlinear oscillations has grown rapidly in the last few decades and has become an area calling for separate and massive treatment. Landau and Lifshitz in their Sections 27—30 provide a concise introduction to the subject, one in which the Russian contributions have been particularly notable. Only the tip of the iceberg, however, is revealed in the discussion.

L. MEIROVITCH, *Methods of Analytical Dynamics.* Modern approaches to oscillatory systems are presented here in a manner that doesn't lose touch with practical reality. The subject of the present chapter appears here under the guise of linear autonomous systems, but the emphasis is on whether the motion is stable, something we have presupposed from the start. Both the classical and modern (e.g., Liapunov method) criteria are discussed. Parametric oscillations (nonautonomous systems) get a separate chapter.

Y. CHEN, *Vibrations: Theoretical Methods.* Much of this engineering-oriented text is concerned with introducing the basic ideas of mechanics, on the one hand, and in treating vibrations of continuous systems on the other. The rest covers the field of the present chapter with many examples worked out and the elements of the pertinent matrices explicitly (and sometimes clumsily) presented. Of particular interest is the application of Laplace and Fourier transform methods for handling problems of forced or driven oscillation.

H. C. CORBEN and **P. STEHLE**, *Classical Mechanics.* The brief chapter on small oscillations in this well-known text concentrates on a number of examples rather than general theory. Special mention should be made of Appendix 3, which gives a brief introduction to the use of group-theoretical methods to reduce the complexity of the eigenfrequency problem for molecules, based on the intrinsic symmetries displayed by the systems. In the precomputer age when problems had to be solved by hand, any method for reducing the amount of computation was naturally the subject of intense concern and study. The advent of the high-speed computer has lessened the calculational importance of these approaches, although the use of symmetry properties to identify zero or degenerate frequencies is of obvious interest. See in this connection

the paper by W. D. Gwinn: "Normal Coordinates: General Theory, Redundant Coordinates, and General Analysis Using Electronic Computers," *Jour. Chem. Phys.* 55, 477 (July 15, 1971).

G. HERZBERG, *Infrared and Roman Spectra of Polyatomic Molecules.* This treatise provides many illustrations of the application of classical small vibration theory to molecular structure. The techniques of using constants of the motion and symmetry properties to reduce the complexity of the calculation are applied here to find explicit solutions for many molecular models. The various normal modes are shown diagrammatically for many of the molecules.

E. **B. WILSON** JR., **J. C.DECIUS**, and **P. C. CROSS**, *Molecular Vibrations.* This 1955 treatise apparently remains the standard treatment of the molecular vibration problems for chemists. It is entirely precomputer in spirit—the long chapter on the benzene molecule ends with the reader adjured to gain facility in the matrix manipulation by checking the calculations of the chapter on a "standard desk-type computing machine." The use of symmetry groups is gone into in considerable detail. Some knowledge of quantum mechanics is needed.

LORD RAYLEIGH, *Theory of Sound.* One of the classics of physics literature, this treatise contains a wealth of theorems and physical illustrations on all of the aspects of vibration theory. Rayleigh himself was responsible for developing much of the theory, especially the introduction of the dissipation function. His treatment is smooth-flowing and clear and contains rarely discussed topics, as on the effects of constraints and the stationary properties of the eigenfrequencies. Rayleigh leans heavily on the work of Routh, who in his Adams Prize Essay of 1877 and in his text *Rigid Dynamics* was one of the first to give a systematic discussion of small vibrations.

E. A. **GUILLEMIN,** *The Mathematics of Circuit Analysis.* This reference is included to indicate the importance of small vibration theory in modern electrical engineering. Considerable attention is paid to quadratic forms and their principal axis transformations. The treatment, which makes abundant use of matrix algebra, is advanced and elegant.