SUGGESTED REFERENCES

P. H. BADGER, *Equilibrium Thermodynamics*. Many textbooks on thermodynamics give some mention of the application of the Legendre transformation to the thermodynamic potentials. This reference has an exceptionally long and explicit treatment in Chapter 10.

R. COURANT AND D. HILBERT, *Methods of Mathematical Physics*. As has been mentioned the Legendre transformation has formal mathematical applications to the theory of partial differential equations. Courant, in Vol. II of this work, provides a readable discussion of this aspect and of the geometrical significance of the Legendre transformation (Chapter 1, Section 6).

E. T. Whittaker, Analytical Dynamics. The subject of the variational principles encountered in classical mechanics can become quite involved and possesses many far-flung roots in apparently unrelated fields. For example, there is a close connection between Hamilton's principle and the general theory of second order partial differential equations. Some of these topics we will discuss in the following chapters, but many are inappropriate for the treatment intended here. Similarly, such questions as whether the extremum in Hamilton's principle is a minimum or maximum cannot be taken up here, nor will it be feasible (or desirable) to consider the many subspecies of variational principles. The student interested in problems of this nature will find an abundant literature; indeed, there is an "embarras des richesses." Only a small fraction of the available references can be mentioned in this list, and of them Whittaker is one of the chief sources. Chapter IX and the first two sections of Chapter X are the portions pertinent to this chapter. The last four sections of Chapter VII are also a rich source of material and examples on oscillations about steady motion although, as often, he makes it seem more complicated than it is.

C. LANCZOS, *Variational Principles of Mechanics*. Noteworthy in Chapter 6 on the canonical equations is the emphasis that the shift to the Hamiltonian formulation doubles the dimensionality of the variable domain – from configuration to phase space. The principle of least action gets a somewhat different viewpoint from what's given here, in his Chapter 5.

J. L. SYNGE, *Classical Dynamics*. This is a respectable-size book (224 pages) tucked away as an article in Vol III/1 of the *Encyclopedia of Physics*. It covers much of the usual topics, often from a highly original viewpoint. Section E on general dynamical theory builds the formulation about the various kinds of variable spaces used, including phase space enlarged by time and energy. Often tough going, but worth the effort.

L. A. PARS, *Treatise on Analytical Dynamics*. Hamilton's equations get a mention only as a section in a chapter on "Further applications of Lagrange's equations." But the material on Routh's method applied to vibrations about steady motion (in Chapter 9) is voluminous and valuable, if somewhat disorganized.

K. R. SYMON, *Mechanics*. The discussion on small vibrations about steady motion (Section 12.6) is somewhat hampered by a refusal to mention the Routhian, but the author speaks on the subject with the voice of the expert. His research interests have included the theory of accelerators, where stability of small oscillations is an important question. Betatron oscillations

are discussed as one example. Another unusual example is the stability of the so-called Lagrangian points in the three-body problem.

D. A. WELLS, *Lagrangian Dynamics*. There *is* a discussion of Hamilton's equations but it is brief and not too informative. However there is an extensive chapter on oscillations about steady motion from the Routhian point of view, replete with figures and examples, mostly with an engineering orientation.

P. A. M. DIRAC, *Lectures on Quantum Mechanics*. A slim book, reprinting some lectures he gave, reporting on his concern with the transition from classical to quantum mechanics. He outlines his method of dealing with the homogeneous problem, for which he devised a hierarchy of weak and strong constraints. Some acquaintance with canonical transformations (given in the next chapter) would be helpful in understanding the material.

H, RUND, *Hamilton-Jacobi Theory in the Calculus of Variations*. Mention was made in the previous chapter of the extensive treatment given here of the homogenous problem, although the approach is not one that seems to have been generally accepted.

D. TER HAAR, *Elements of Hamiltonian Mechanics*. Reasons have been given above (p. 364) for the disagreement with the author's treatment of the modified Hamilton's principle. What is noteworthy, however, is the treatment of variational principles where time is varied, as in the principle of least action, and the formal extension of time and energy as canonical variable – although the dangers inherent in the homogeneous problem get only scant warning.

P. BRUNET, *Étude Historique sur le Principe de la Moindre Action*. Those interested in the early history of the principle of least action will find it here in a smooth urbane treatment, from the teleological beginnings of Maupertuis to the time of Lagrange when it had been transformed into a solid tool for developing mechanics.

R. L. LINDSAY AND H. MARGENAU, *Foundations of Physics*. The statements of Hamilton's principle and the principle of least action appear to endow the mechanical system with conscious knowledge of the final state toward which the motion is directed. Such an appearance is of course illusory; the motion of the system is determined only by the initial conditions. But the view has given rise in the past to much philosophical speculation. Chapter 3 of this text presents an adequate discussion of this and similar points, and furnishes references for further reading for those inclined.