

SUGGESTED REFERENCES

C. CARATHÉODORY, *Calculus of Variations and Partial Differential Equations*. Canonical or contact transformations were first introduced by mathematicians in the theory of partial differential equations. Many of the properties of the transformations that physicists use were first developed for these purposes. The most thorough exposition of these mathematical origins from which the physics applications came is in Carathéodory's masterful treatise. Fortunately it is now available in English translation. The pertinent references are in Chapters 4-7 of Vol. 1 on partial differential equations. They contain, among many other topics, a fuller proof of what is referred to above (p. 403) as "Carathéodory's Theorem." One keeps coming back to Carathéodory time after time, to see how "it's done properly."

H. RUND, *Hamilton-Jacobi Theory in the Calculus of Variations*. This reference provides a somewhat different mathematical picture (in Chapter 2, Sections 12-14) with the physical applications kept clearly in mind. For much of the discussion, time is considered one of the canonical variables.

M. BORN, *The Mechanics of the Atom*. The subject of the canonical transformations of classical mechanics played an important role in the first formulations of both the older Bohr quantum theory and the newer quantum mechanics. Hence many treatises ostensibly devoted to one or the other of these forms of quantum mechanics often contain detailed expositions of the needed branches of classical mechanics. Outstanding among them is this 1924 volume of Born, written before the days of wave mechanics. The first chapter succinctly discusses canonical transformations and gives many interesting physical illustrations. There is no mention of Poisson brackets for they became of special interest to the modern physicist only with the advent of Heisenberg's and Dirac's formulation of quantum mechanics.

R. C. TOLMAN, *The Principles of Statistical Mechanics*. A veritable encyclopedia of theoretical physics, Chapter II of this bulky volume gives a brief but clear discussion of canonical transformations and similar topics in classical mechanics. The properties of Poisson brackets are included in the treatment. Section 19, Chapter III, is concerned with Liouville's theorem.

C. LANZOS, *The Variational Principles of Mechanics*. Canonical transformations entered into mechanics first through perturbation theory in classical mechanics, indeed long before it was realized quite what they were. (References on these applications will be given below in Chapter 11 on perturbation theory.) By now any text on mechanics above the intermediate level devotes considerable attention to the subjects of Poisson bracket formulations and canonical transformations. Only a few references can be cited explicitly. Lanczos has a different viewpoint from others; he talks a good deal and writes relatively few equations. The subject is tied into its mathematical origins, but the use is physical.

L. PARS, *A Treatise on Analytical Dynamics*. Canonical transformations are made use of in this text a number of times before they are explicitly named and discussed (as contact transformations) in Chapter 24. Both the symplectic and the generator approaches are included. Particularly noteworthy is a proof in effect of Carathéodory's theorem that explicitly leads to the generating function.

J. L. SYNGE, *Classical Dynamics*. For the most part Synge works in a phase space that includes t and its conjugate momentum H , but otherwise the approach has considerable similarities with that adopted here. The symplectic method is given in matrix form, although the notation is quite different. A method is given for going from the symplectic to the generating function approach but the path is not clear or always convincing.

C. W. KILMISTER, *Hamiltonian Dynamics*. For those equipped with a modern mathematical background in tensor analysis and differential geometry, this little book presents an elegant and compact discussion with a heavy emphasis on the symplectic approach. The author presents a “universal generator” in η and ζ from whence the four types may be derived.

E. J. SALETAN AND A. H. CROMER, *Theoretical Mechanics*. Considerable space is given to canonical transformations mostly from the symplectic viewpoint, but including the transition to the generating function approach. Canonical is used in the sense of what is here called extended canonical transformation. In addition there is added the highly unorthodox, if not downright dangerous, notion of a *canonoid* transformation – one that is canonical only for certain types of Hamiltonians. (Most applications of canonical transformations depend on the property that they be canonical for *all* Hamiltonians.) Canonical transformations depending on a continuous parameter are discussed explicitly.

E. C. G. SUDARSHAN AND N. MUKUNDA, *Classical Dynamics*. This is a treatment of mechanics permeated with a group-theoretical approach. It might be termed classical mechanics as viewed by theoretical-particle physicists. Much of the book is concerned indeed with canonical transformations and the implications of the symmetries of the system and the transformations.

H. V. MCINTOSH, *Symmetry and Degeneracy*, in *Group Theory and its Applications*, vol. II, E. M. Loebl, ed. Reference is made again to this enthusiastic survey of the connections between symmetries of the system and the constants of the motion. While canonical transformations per se are rarely mentioned, the notion that the generators of symmetry operations provide the constants of motion appears quite frequently, and many examples are provided.