Anatomy of double-beta-decay nuclear matrix elements

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The status of the present knowledge of the neutrino oscillation phenomena is schematically depicted in this slide. Three quantities are unknown at present:

a) The mass $m_1$

b) The angle $\theta_{13}$

c) Whether the normal or inverted hierarchy is realized.
However, $\nu$ masses are much smaller than the masses of other fermions.

Is that a possible "Hint of" a new mass-generating mechanism?
The most popular theory of why neutrinos are so light is the —

See-Saw Mechanism

(Gell-Mann, Ramond, Slansky (1979), Yanagida(1979), Mohapatra, Senjanovic(1980))

It assumes that the very heavy neutrinos $N_R$ exist. Their mass plays an analogous role as the scale $\Lambda$ of Weinberg, i.e., $m_\nu \sim \nu^2/M_N$. Both the light and heavy neutrinos are Majorana fermions.
What is the relation of the deduced fundamental parameters and the neutrino mixing matrix? Or, in other words, what is the relation between the $0\nu\beta\beta$ decay rate and the absolute neutrino mass?

As long as the mass eigenstates $\nu_i$ are Majorana neutrinos, the $0\nu\beta\beta$ decay will occur, with the rate

$$1/T_{1/2} = G(E_{\text{tot}}, Z) (M^{0\nu})^2 \langle m_{\beta\beta} \rangle^2,$$

where $G(E_{\text{tot}}, Z)$ is easily calculable phase space factor, $M^{0\nu}$ is the nuclear matrix element, calculable with difficulties (and discussed later), and

$$\langle m_{\beta\beta} \rangle = \sum_i |U_{ei}|^2 \exp(i\alpha_i) m_i,$$

where $\alpha_i$ are unknown Majorana phases (only two of them are relevant). We can relate $\langle m_{\beta\beta} \rangle$ to other observables related to the absolute neutrino mass.
from observational cosmology, $M = m_1 + m_2 + m_3$

blue shading: normal hierarchy, $\Delta m_{31}^2 > 0$.
red shading: inverted hierarchy $\Delta m_{31}^2 < 0$

shading: best fit parameters, lines 95% CL errors.

Thanks to A. Piepke
Nuclear matrix elements

A provocative question: Do we know at all how large the matrix elements really are? Or, in other words, why there is so much variation among the published calculated matrix elements?

This suggests an uncertainty of as much as a factor of 5. Is it really so bad?
Why it is difficult to calculate the matrix elements accurately?

Contributions of different angular momenta $J$ of the neutron pair that is transformed in the decay into the proton pair with the same $J$.

Note the opposite signs, and thus tendency to cancel, between the $J = 0$ (pairing) and the $J \neq 0$ (ground state correlations) parts.

The same restricted s.p. space is used for QRPA and NSM. There is a reasonable agreement between the two methods.
The opposite signs, and similar magnitudes of the $J = 0$ and $J \neq 0$ parts is universal. Here for three nuclei with coupling constant $g_{pp}$ adjusted so that the $2\nu\beta\beta$ rate is correctly reproduced. Now two oscillator shells are included.
Contributions to the $0\nu\beta\beta$-decay NME in $^{100}$Mo from different $J^{\pi,+}$ multipole states in the virtual intermediate odd-odd $^{100}$Tc.

The strength $g_{pp}$ of the particle-particle neutron-proton interaction is varied by ± 5% with respect to the value $g_{pp} = 1.096$ that gives good agreement with the known $2\nu\beta\beta$ decay rate.

Note the rapid variation with $g_{pp}$ of the $1^{+}$ multipole. The other multipoles (as well as the negative parity ones not shown) change only slowly with $g_{pp}$.

It is therefore reasonable to adjust $g_{pp}$ so that the contribution of the $1^{+}$ is fixed to the known $2\nu\beta\beta$ where only that multipole contributes. This is what we do.
Calculated values of Nuclear matrix elements for $^{76}\text{Ge}$.

- **BKS-01** = A. Bobyk, W. Kaminski, F. Simkovic, *PRC* **63** (2001) \(\Rightarrow 2\nu\beta\beta\) 20 times too slow
- **TFSG-86** = T. Tomoda, A. Faessler, K. W. Schmid, F. Grummer, *NPA* **452** (1986) \(\Leftarrow 2\nu\beta\beta\) 8 times too fast
Dependence on the relative distance, nucleon structure, short range repulsion, higher order currents, etc.

The neutrino propagator connecting the two participating nucleons introduces dependence on the relative distance $r$ (or equivalently momentum transfer $q$) between them.

If small values of $r$ (or large values of $q$) are important, we have to worry about induced weak currents, nucleon finite size, and the short range nucleon-nucleon repulsion.
Graphs representing the elementary $0\nu\beta\beta$ amplitude. The neutrino propagator causes dependence of the corresponding transition operator on the momentum transfer $q$ and, in the Fourier transform on the distance $r$ between the participating nucleons.

The "neutrino potential" is $H(r) = R/r \Phi(\omega r)$, where $\Phi(\omega r)$ is rather slowly varying function. Thus, naively, one expects that the typical distance is $r \sim R$. 
The radial dependence of $M^{0\nu}$ for the three indicated nuclei. The contributions summed over all components $ss$ shown in the upper panel. The `pairing' $J = 0$ and `broken pairs' $J \neq 0$ parts are shown separately below. Note that these two parts essentially cancel each other for $r > 2$-3 fm. This is a generic behavior. Hence the treatment of small values of $r$ and large values of $q$ are quite important.

$$M^{0\nu} = \int C(r) dr$$
The radial dependence of $M^{0\nu}$ for the indicated nuclei, evaluated in the nuclear shell model. (Menendes et al, arXiv:0801.3760).

Note the similarity to the QRPA evaluation of the same function.
Dependence on the distance between the two transformed nucleons and the effect of different treatments of short range correlations. This causes changes of $M^{0\nu}$ by $\sim 20\%$. 

Graph by F. Simkovic
Dependence of the $0\nu\beta\beta$ matrix element on the $M_A = M_V = \Lambda_{\text{cut}}$ parameter in the usual dipole nucleon form factor. When correction for short range correlations is included the $M^{0\nu}$ changes little for $\Lambda_{\text{cut}} \geq 1000$ MeV.

Graph by F. Simkovic

\[ f(q^2) \sim \frac{1}{(1+q^2/\Lambda^2)^2} \]
Contributions of different parts of the nucleon current. Note that the AP (axial-pseudoscalar interference) contains $q^2/(q^2 + m_\pi^2)$, and MM contains $q^2/4M_p^2$. 

\[ 76\text{Ge} \rightarrow 76\text{Se} \]
Full estimated range of $M^{0\nu}$ within QRPA framework and comparison with NSM (higher order currents now included in NSM)
Conclusions:

• Various physics effects that influence the magnitude of the $0\nu\beta\beta$ nuclear matrix elements have been identified.
• The corresponding corrections, within QRPA, were estimated.
• In particular, the competition between the `pairing', $J = 0$, and the `broken pairs', $J \neq 0$, contributions causes almost complete cancellation for the internucleon distance $r \geq 2-3$ fm, hence making the short range behavior important.
• Thus the treatment of the nucleon finite size, induced weak currents and the short range nucleon-nucleon repulsion causes visible changes in the nuclear matrix elements.
• There is little independent information about such effects (for analogous charge-changing operators). Thus, the prudent approach is to include them in the corresponding systematic error.
• The total range, assuming the basic validity of QRPA, is reasonable, and the qualitative agreement with the ISM is encouraging.

Results obtained in collaboration with Fedor Simkovic, Vadim Rodin, Amand Faessler and Jonathan Engel.
Spares
How good is the closure approximation?

Comparison between the QRPA $M^{0\nu}$ with the proper energies of the virtual intermediate states (symbols with arrows) and the closure approximation (lines) with different $<E_n - E_i>$. Note the mild dependence on $<E_n - E_i>$ and the fact that the exact results are reasonably close to the closure approximation results for $<E_n - E_i> < 20$ MeV.
Contribution of different momentum transfers $q$ to the $0\nu\beta\beta$ matrix element in $^{76}\text{Ge} \rightarrow ^{76}\text{Se}$ decay.

Here the curves peak at $q \sim 100$ MeV, with a long tail extending to $\sim 500$ MeV.

$M^{0\nu} = \int C(q) dq$
Competition between the $J = 0$ and $J \neq 0$ parts as a function of momentum transfer $q$. Note the change of scale compared to the previous slide.

These curves peak at $\sim 40$ MeV. For $q > 200$ MeV the $J = 0$ part is dominating.
Study of the $0\nu\beta\beta$-decay is one of the highest priority issues in particle and nuclear physics

(physics/0411216)

We recommend, as a high priority, a phased program of sensitive searches for neutrinoless double beta decay (first on the list of recommendations)

The answer to the question whether neutrinos are their own antiparticles is of central importance, not only to our understanding of neutrinos, but also to our understanding of the origin of mass.